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Theory of Complex Variables - MA 209
Problem Sheet - 6
Complex Integration

1. Evaluate the the given integral along the indicated contour.

   (a) $\int_C \frac{z+1}{z} dz$ where C is the right half of the circle $|z| = 1$ from $z = -i$ to $z = i$
   (b) $\int_C Re(z) dz$ where C is the circle $|z| = 1$
   (c) $\int_C x^2 + iy^3 dz$ where C is the straight line from $z = 1$ to $z = i$
   (d) $\int_C x^2 - iy^3 dz$ where C is the lower half of the circle $|z| = 1$ from $z = -1$ to $z = 1$

2. Find an upper bound for the absolute value of the given integral along the indicated contour. $\int_C \frac{e^z}{z^2+1} dz$ where C the circle $|z| = 5$

3. Show that $\int_C f(z) dz = 0$ for the following f; C is the unit circle $|z| = 1$
   (a) $f(z) = z^2 + \frac{1}{z-4}$
   (b) $f(z) = \frac{z-3}{z^2+2z+2}$
   (c) $f(z) = \frac{e^z}{2z^2+11z+15}$

4. Evaluate the integral $\int_C (\frac{e^z}{z+3} - 3\bar{z}) dz$ where C the circle $|z| = 1$

5. Suppose z_0 is any constant complex number inside to any simple closed curve C. Show that for a positive integer $n > 1$, $\int_C \frac{1}{(z-z_0)^n} dz = 0$

6. Evaluate $\int_C (z^3 + +z^2 + Re(z)) dz$ where C the triangle with the vertices $0, 1 + 2i$ and 1.

7. Describe contours C for which we are guaranteed that $\int_C f(z) dz = 0$ for each of the following functions
   (a) $f(z) = \frac{1}{z^3+z}$
   (b) $f(z) = \frac{1}{1-e^z}$
   (c) $f(z) = Ln(z)$

8. Evaluate the given integral along the indicated contours.
   (a) $\int_C \frac{4}{z-3i} dz : |z| = 5$
   (b) $\int_C \frac{e^z}{z-\pi i} dz : |z| = 4$
   (c) $\int_C \frac{1+e^z}{z} dz : |z| = 1$
   (d) $\int_C \frac{z^2-3z+4i}{z+2i} dz : |z| = 3$

9. Evaluate the following integrals

   (a) $\int_C \frac{1}{z} dz$ where C is the arc of the circle $z = 4e^{it}$ $\frac{-\pi}{2} \leq t \leq \frac{\pi}{2}$
   (b) $\int_C \frac{1}{z} dz$ where C is the line segment from $1 + i$ to $4 + 4i$

10. Evaluate the integral $\int_C Im(z - i) dz$ where C is the polygonal path consisting of the circular arc along $|z| = 1$ from $z = 1$ to $z = i$ and the line segment from $z = i$ to $z = -1$

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